EXERCISE 1.1

1. Use Euclid's division algorithm to find the HCF of :
   (i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255
2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.
3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

   [**Hint :** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]
5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

1.3 The Fundamental Theorem of Arithmetic

In your earlier classes, you have seen that any natural number can be written as a product of its prime factors. For instance, $2 = 2$, $4 = 2 \times 2$, $253 = 11 \times 23$, and so on. Now, let us try and look at natural numbers from the other direction. That is, can any natural number be obtained by multiplying prime numbers? Let us see.

Take any collection of prime numbers, say 2, 3, 7, 11 and 23. If we multiply some or all of these numbers, allowing them to repeat as many times as we wish, we can produce a large collection of positive integers (In fact, infinitely many). Let us list a few :

$7 \times 11 \times 23 = 1771$ $\quad$ $3 \times 7 \times 11 \times 23 = 5313$

$2 \times 3 \times 7 \times 11 \times 23 = 10626$ $\quad$ $2^3 \times 3 \times 7^3 = 8232$

$2^2 \times 3 \times 7 \times 11 \times 23 = 21252$

and so on.

Now, let us suppose your collection of primes includes all the possible primes. What is your guess about the size of this collection? Does it contain only a finite number of integers, or infinitely many? Infact, there are infinitely many primes. So, if we combine all these primes in all possible ways, we will get an infinite collection of numbers, all the primes and all possible products of primes. The question is – can we produce all the composite numbers this way? What do you think? Do you think that there may be a composite number which is not the product of powers of primes? Before we answer this, let us factorise positive integers, that is, do the opposite of what we have done so far.